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International Journal of Solids and Structures 42 (2005) 3171–3183
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Amplification of acoustic waves in piezoelectric semiconductor plates

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Received 6 October 2004
Available online 18 November 2004

Abstract

Two-dimensional equations for coupled extensional, flexural and thickness-shear motions of thin plates of piezoelectric semiconductors are obtained systematically from the three-dimensional equations by retaining lower order terms in power series expansions in the plate thickness coordinate. The two-dimensional equations are specialized to crystals of 6 mm symmetry and are simplified by thickness-shear approximation. Propagation of thickness-shear waves and their amplification by a dc electric field are analyzed.

Keywords: Piezoelectric; Semiconductor; Plate

1. Introduction

Piezoelectric materials are either dielectrics or semiconductors (Auld, 1973). An acoustic wave propagating in a piezoelectric crystal is usually accompanied by an electric field. When the crystal is also semiconducting, the electric field produces currents and space charge resulting in dispersion and acoustic loss (Hutson and White, 1962). The interaction between a traveling acoustic wave and mobile charges in piezoelectric semiconductors is called the acoustoelectric effect which is a special case of a more general phenomenon which may be called wave-particle drag (Weinreich et al., 1959). It was also found that an acoustic wave traveling in a piezoelectric semiconductor can be amplified by application of an initial or biasing dc electric field (White, 1962). Acoustoelectric effect and acoustoelectric amplification of acoustic waves have led to many acoustoelectric devices, e.g., Kino (1976), Heyman (1978), Busse and Miller (1981), and Dietz et al. (1988). The basic behavior of piezoelectric semiconductors and the acoustoelectric effect

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doi:10.1016/j.ijsolstr.2004.10.011

can be described by a linear phenomenological theory (Hutson and White, 1962; White, 1962). More sophisticated nonlinear theories for deformable semiconductors have also been developed (de Lorenzi and Tiersten, 1975; Maugin and Daher, 1986).

Piezoelectric devices, dielectrics or semiconductors, often have structural shapes of single or multi-layered plates, or plates on substrates. Two-dimensional equations for thin piezoelectric dielectric plates have been developed (Mindlin, 1972; Lee et al., 1987; Tiersten, 1993; Yong et al., 1993) and proved very effective in device modeling (Wang and Yang, 2000). In this paper we study motions of thin plates of piezoelectric semiconductors. The three-dimensional equations of linear piezoelectric semiconductors are summarized in Section 2. Two-dimensional equations for thin plates are derived systematically from the three-dimensional equations in Section 3. The equations are specialized to crystals of 6 mm symmetry in Section 4. Propagation of thickness-shear waves under a dc field is analyzed in Section 5. Finally, some conclusions are drawn in Section 6.

2. Three-dimensional equations

Consider a homogeneous, one-carrier piezoelectric semiconductor under a uniform dc electric field $\overline{E}_j$. The steady state current is $\overline{J}_i = q\bar{n}\mu_{ij}\overline{E}_j$, where q is the carrier charge, $\bar{n}$ is the steady state carrier density which produces electrical neutrality, and μ_{ij} is the carrier mobility. The summation convention for repeated tensor indices is used. When an acoustic wave propagates through the material, perturbations of the electric field, the carrier density and the current are denoted by E_j, n and J_i. The linear theory for small signals (Hutson and White, 1962; Wauer and Suherman, 1997) consists of the equations of motion, Gauss's law of electrostatics, and the conservation of charge

$$\begin{aligned} &T_{ji,j} = \rho\ddot{u}_i, \\ &D_{i,i} = qn, \\ &q\dot{n} + J_{i,i} = 0, \end{aligned} \tag{1}$$

where u_i is the displacement vector, T_{ij} the stress tensor, ρ the mass density, and D_i the electric displacement vector. A comma followed by an index denotes partial differentiation with respect to the coordinate associated with the index. A superimposed dot represents differentiation with respect to time t. The above equations are accompanied by the following constitutive relations (White, 1962):

$$\begin{aligned} &T_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k, \\ &D_i = e_{ijk}S_{jk} + \varepsilon_{ij}E_j, \\ &J_i = q\bar{n}\mu_{ij}E_j + qn\mu_{ij}\overline{E}_j - qd_{ij}N_j, \end{aligned} \tag{2}$$

where the strain tensor S_{ij}, the electric potential ϕ, and the carrier density gradient N_j are defined by

$$\begin{aligned} &S_{ij} = (u_{i,j} + u_{j,i})/2, \\ &E_i = -\phi_{,i}, \\ &N_j = n_{,j}. \end{aligned} \tag{3}$$

In (2), c_{ijkl}, e_{kij} and ε_{ij} are the elastic, piezoelectric and dielectric constants. d_{ij} are the carrier diffusion constants. With successive substitutions form (2) and (3), (1) can be written as five equations for $\mathbf{u}$, ϕ and n

$$\begin{aligned} &c_{ijkl}u_{k,lj} + e_{kij}\phi_{,kj} = \rho\ddot{u}_i, \\ &e_{ikl}u_{k,li} - \varepsilon_{ij}\phi_{,ij} = qn, \\ &\dot{n} - \bar{n}\mu_{ij}\phi_{,ij} + \mu_{ij}\overline{E}_j n_{,i} - d_{ij}n_{,ij} = 0. \end{aligned} \tag{4}$$

On the boundary of a finite body with a unit outward normal n_i, usually the mechanical displacement u_i or the traction vector $T_{ij}n_i$, the electric potential ϕ or the normal component of the electric displacement vector $D_i n_i$, and the carrier density n or the normal current $J_i n_i$ are prescribed (Wauer and Suherman, 1997).

3. Derivation of two-dimensional plate equations

Consider a piezoelectric semiconductor plate of thickness $2h$ as shown in Fig. 1. Part of the plate major surfaces may be electroded. The electrodes are assumed to be very thin and their mechanical effects will be neglected as in Mindlin (1972), Lee et al. (1987), Tiersten (1993) and Yong et al. (1993). For a first-order plate theory of coupled extensional, flexural and thickness-shear motions we make the following expansions of the mechanical displacement, electric potential and carrier density:

$$\begin{aligned} u_a(x_1,x_2,x_3,t) &\cong u_a^{(0)}(x_1,x_2,t) + x_3 u_a^{(1)}(x_1,x_2,t), \quad a = 1,2, \\ u_3(x_1,x_2,x_3,t) &\cong u_3^{(0)}(x_1,x_2,t) + x_3 u_3^{(1)}(x_1,x_2,t) + x_3^2 u_3^{(2)}(x_1,x_2,t), \\ \phi(x_1,x_2,x_3,t) &\cong \phi^{(0)}(x_1,x_2,t) + x_3 \phi^{(1)}(x_1,x_2,t), \\ n(x_1,x_2,x_3,t) &\cong n^{(0)}(x_1,x_2,t) + x_3 n^{(1)}(x_1,x_2,t), \end{aligned} \tag{5}$$

where we have introduced another convention that the indices a and b assume 1 and 2 only but not 3. $u_a^{(0)}$ are the plate extensional displacements, $u_3^{(0)}$ the flexural displacement, and $u_a^{(1)}$ the thickness-shear displacements. $u_3^{(1)}$ and $u_3^{(2)}$ are the plate thickness stretch displacements accompanying extension and flexure, which will be eliminated later through a stress relaxation condition. Substitution of (5) into (3) results in the following expressions of the strain, electric field and carrier density gradient:

$$\begin{aligned} S_p &\cong S_p^{(0)} + x_3 S_p^{(1)}, \quad p = 1,2,\ldots,6, \\ E_i &\cong E_i^{(0)} + x_3 E_i^{(1)}, \\ N_i &\cong N_i^{(0)} + x_3 N_i^{(1)}, \end{aligned} \tag{6}$$

where, under the compact matrix notation (Tiersten, 1969a), the indices p and q range from 1 to 6. The zero and first order strains are defined as

$$\begin{aligned} &S_1^{(0)} = u_{1,1}^{(0)}, \quad S_2^{(0)} = u_{2,2}^{(0)}, \quad S_3^{(0)} = u_3^{(1)}, \\ &S_4^{(0)} = u_{3,2}^{(0)} + u_2^{(1)}, \quad S_5^{(0)} = u_{3,1}^{(0)} + u_1^{(1)}, \quad S_6^{(0)} = u_{1,2}^{(0)} + u_{2,1}^{(0)}, \end{aligned} \tag{7}$$

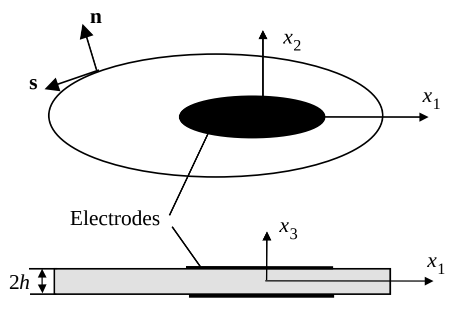

Fig. 1. Plan view and cross section of a thin plate of piezoelectric semiconductor.

and

$$\begin{aligned} &S_1^{(1)} = u_{1,1}^{(1)}, \quad S_2^{(1)} = u_{2,2}^{(1)}, \quad S_3^{(1)} = 2u_3^{(2)}, \\ &S_4^{(1)} = 0, \quad S_5^{(1)} = 0, \quad S_6^{(1)} = u_{1,2}^{(1)} + u_{2,1}^{(1)}. \end{aligned} \tag{8}$$

We note that $S_3^{(0)}$ and $S_3^{(1)}$ are involved with $u_3^{(1)}$ and $u_3^{(2)}$ which are to be eliminated later. The zero and first order electric fields are given by

$$E_1^{(0)} = -\phi_{,1}^{(0)}, \quad E_2^{(0)} = -\phi_{,2}^{(0)}, \quad E_3^{(0)} = -\phi^{(1)}, \tag{9}$$

and

$$E_1^{(1)} = -\phi_{,1}^{(1)}, \quad E_2^{(1)} = -\phi_{,2}^{(1)}, \quad E_3^{(1)} = 0. \tag{10}$$

The zero and first order gradients of the carrier density are

$$N_1^{(0)} = n_{,1}^{(0)}, \quad N_2^{(0)} = n_{,2}^{(0)}, \quad N_3^{(0)} = n^{(1)}, \tag{11}$$

and

$$N_1^{(1)} = n_{,1}^{(1)}, \quad N_2^{(1)} = n_{,2}^{(1)}, \quad N_3^{(1)} = 0. \tag{12}$$

Integrating the equations in (1) and their products with x_3 through the plate thickness respectively, we obtain the following two-dimensional equations of motion, Gauss's law and conservation of charge:

$$\begin{aligned} &T_{ab,a}^{(0)} + t_b^{(0)} = 2h\rho\ddot{u}_b^{(0)}, \\ &T_{a3,a}^{(0)} + t_3^{(0)} = 2h\rho\ddot{u}_3^{(0)}, \\ &T_{ab,a}^{(1)} - T_{3b}^{(0)} + t_b^{(1)} = \frac{2h^3}{3}\rho\ddot{u}_b^{(1)}, \\ &D_{a,a}^{(0)} + d^{(0)} = 2hqn^{(0)}, \\ &D_{a,a}^{(1)} - D_3^{(0)} + d^{(1)} = \frac{2h^3}{3}qn^{(1)}, \\ &2hq\dot{n}^{(0)} + J_{a,a}^{(0)} + j^{(0)} = 0, \\ &\frac{2h^3}{3}q\dot{n}^{(1)} + J_{a,a}^{(1)} - J_3^{(0)} + j^{(1)} = 0, \end{aligned} \tag{13}$$

where the plate resultants and surface loads of various orders are defined by

$$\begin{aligned} &\{T_{ij}^{(n)}, D_i^{(n)}, J_i^{(n)}\} = \int_{-h}^{h} x_3^n \{T_{ij}, D_i, J_i\}\,\mathrm{d}x_3, \\ &t_j^{(n)} = [x_3^n T_{3j}]_{-h}^{h}, \quad d^{(n)} = [x_3^n D_3]_{-h}^{h}, \\ &j^{(n)} = [x_3^n J_3]_{-h}^{h}, \quad n = 0, 1. \end{aligned} \tag{14}$$

Since the plate is assumed to be thin, we make the stress relaxation approximation of vanishing normal stress $T_{33} = 0$. This implies, through $(2)_1$ by setting $i = j = 3$, the following expression for $u_{3,3}$ in terms of other components of the displacement and potential gradients:

$$u_{3,3} = -\frac{1}{c_{3333}}(c_{33kl}u_{k,l} - c_{3333}u_{3,3} - e_{k33}E_k). \tag{15}$$

We note that stress relaxation for thin anisotropic or piezoelectric plates can be made in ways more sophisticated than the above, also involving T_{31} and T_{32} (Mindlin, 1972). That is not our main interest here. The above relaxation involving T_{33} is the major relaxation because in anisotropic plates couplings among extensions in different directions are usually much stronger than couplings between extensions and shears. We also note that in (15) $u_{3,3}$ has been eliminated on the right-hand side because when $i = j = 3$ the two terms containing $u_{3,3}$ will cancel with each other. From (15) the thickness expansion or contraction accompanying the extension and flexure of the plate due to Poisson's effect can be found if wanted. Substituting (15) back into $(2)_{1,2}$, we obtain the following constitutive relations relaxed for thin plates:

$$\begin{aligned} T_{ij} &= \bar{c}_{ijkl}u_{k,l} - \bar{e}_{kij}E_k, \\ D_i &= \bar{e}_{ikl}u_{k,l} + \bar{\varepsilon}_{ij}E_j, \end{aligned} \tag{16}$$

where the relaxed material constants are defined by

$$\begin{aligned} \bar{c}_{ijkl} &= c_{ijkl} - c_{ij33}c_{33kl}/c_{3333}, \\ \bar{e}_{kij} &= e_{kij} - e_{k33}c_{33ij}/c_{3333}, \\ \bar{\varepsilon}_{ij} &= \varepsilon_{ij} + e_{i33}e_{j33}/c_{3333}. \end{aligned} \tag{17}$$

We note that the right-hand sides of (16) do not contain $u_{3,3}$ and $T_{33} = 0$ is automatically satisfied by (16).

Integrating (16) and $(2)_3$ through the plate thickness, we obtain the zero order constitutive relations

$$\begin{aligned} T_{ij}^{(0)} &= 2h(c_{ijkl}^{(0)}S_{kl}^{(0)} - e_{kij}^{(0)}E_k^{(0)}), \\ D_i^{(0)} &= 2h(e_{ijk}^{(0)}S_{jk}^{(0)} + \bar{\varepsilon}_{ij}E_j^{(0)}), \\ J_i^{(0)} &= 2h(q\bar{n}\mu_{ij}E_j^{(0)} + qn^{(0)}\mu_{ij}\overline{E}_j - qd_{ij}N_j^{(0)}), \end{aligned} \tag{18}$$

where, following Mindlin (1972), we have modified $\bar{c}_{ijkl}$ and $\bar{e}_{ijk}$ to $c_{ijkl}^{(0)}$ and $e_{ijk}^{(0)}$ through the introduction of two shear correction factors κ_1 and κ_2 by the replacement of the following zero order strains:

$$S_{31}^{(0)} \to \kappa_1 S_{31}^{(0)}, \quad S_{32}^{(0)} \to \kappa_2 S_{32}^{(0)}. \tag{19}$$

The two correction factors should be determined by requiring the two fundamental thickness-shear resonant frequencies obtained from the two-dimensional plate equations to be equal to the corresponding exact frequencies predicted by the three-dimensional equations. With shear correction factors thus determined, the two-dimensional plate equations and the exact three-dimensional equations yield the same frequencies for a particular motion, i.e., the thickness-shear vibrations of a plate in the two fundamental thickness-shear modes. Multiplying both sides of (16) and $(2)_3$ by x_3 and integrating the resulting equation through the plate thickness we have the first order constitutive relations

$$\begin{aligned} T_{ij}^{(1)} &= \frac{2h^3}{3}(\bar{c}_{ijkl}S_{kl}^{(1)} - \bar{e}_{kij}E_k^{(1)}), \\ D_i^{(1)} &= \frac{2h^3}{3}(\bar{e}_{ijk}S_{jk}^{(1)} + \bar{\varepsilon}_{ij}E_j^{(1)}), \\ J_i^{(1)} &= \frac{2h^3}{3}(q\bar{n}\mu_{ij}E_j^{(1)} + qn^{(1)}\mu_{ij}\overline{E}_j - qd_{ij}N_j^{(1)}). \end{aligned} \tag{20}$$

In summary, we have obtained the two-dimensional equations of motion, Gauss's law and conservation of charge (13), the constitutive relations (18) and (20), and the displacement, potential and carrier density gradients (7)–(12). With successive substitutions, (13) can be written as nine equations for the nine unknowns of $u_1^{(0)}$, $u_2^{(0)}$, $u_3^{(0)}$, $u_1^{(1)}$, $u_2^{(1)}$, $\phi^{(0)}$, $\phi^{(1)}$, $n^{(0)}$, and $n^{(1)}$. At the boundary of a plate with in-plane unit exterior normal **n** and in-plane unit tangent **s** (Fig. 1), we may prescribe

$$\begin{aligned}&T_{nn}^{(0)} \text{ or } u_n^{(0)}, \quad T_{ns}^{(0)} \text{ or } u_s^{(0)}, \quad T_{n3}^{(0)} \text{ or } u_3^{(0)},\\ &T_{nn}^{(1)} \text{ or } u_n^{(1)}, \quad T_{ns}^{(1)} \text{ or } u_s^{(1)},\\ &D_n^{(0)} \text{ or } \phi^{(0)}, \quad D_n^{(1)} \text{ or } \phi^{(1)},\\ &J_n^{(0)} \text{ or } n^{(0)}, \quad J_n^{(1)} \text{ or } n^{(1)}.\end{aligned} \tag{21}$$

4. Equations for crystals of 6 mm symmetry

Quite a few piezoelectric semiconductors are of 6 mm symmetry. This includes, e.g., widely used beryllium oxide (BeO), cadmium selenide (CdSe), cadmium sulfide (CdS), zinc oxide (ZnO), and zinc sulfide (ZnS) (Auld, 1973). For crystals of 6 mm symmetry, when the 6-fold axis is along the x_3 axis, the material tensors in (2) can be represented by the following matrices under the compact matrix notation (Tiersten, 1969a):

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{66}\end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & e_{31}\\ 0 & 0 & e_{31}\\ 0 & 0 & e_{33}\\ 0 & e_{15} & 0\\ e_{15} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}^{\mathrm{T}}, \quad \begin{pmatrix} \varepsilon_{11} & 0 & 0\\ 0 & \varepsilon_{11} & 0\\ 0 & 0 & \varepsilon_{33}\end{pmatrix}, \tag{22}$$

where $c_{66} = (c_{11} - c_{12})/2$, and the superscript 'T' indicates matrix transpose. μ_{ij} and d_{ij} have the same structure as ε_{ij}. The constitutive relations take the following form:

$$\begin{aligned}T_{11}^{(0)} &= 2h(\bar{c}_{11}u_{1,1}^{(0)} + \bar{c}_{12}u_{2,2}^{(0)} + \bar{e}_{31}\phi^{(1)}),\\ T_{22}^{(0)} &= 2h(\bar{c}_{12}u_{1,1}^{(0)} + \bar{c}_{11}u_{2,2}^{(0)} + \bar{e}_{31}\phi^{(1)}),\\ T_{3a}^{(0)} &= 2h[\kappa^2 c_{44}(u_{3,a}^{(0)} + u_a^{(1)}) + \kappa e_{15}\phi_{,a}^{(0)}],\\ T_{12}^{(0)} &= 2hc_{66}(u_{1,2}^{(0)} + u_{2,1}^{(0)}),\end{aligned} \tag{23}$$

$$\begin{aligned}T_{11}^{(1)} &= \frac{2}{3}h^3(\bar{c}_{11}u_{1,1}^{(1)} + \bar{c}_{12}u_{2,2}^{(1)}),\\ T_{22}^{(1)} &= \frac{2}{3}h^3(\bar{c}_{12}u_{1,1}^{(1)} + \bar{c}_{11}u_{2,2}^{(1)}),\\ T_{12}^{(1)} &= \frac{2}{3}h^3 c_{66}(u_{1,2}^{(1)} + u_{2,1}^{(1)}),\end{aligned} \tag{24}$$

$$\begin{aligned}D_a^{(0)} &= 2h[\kappa e_{15}(u_{3,a}^{(0)} + u_a^{(1)}) - \varepsilon_{11}\phi_{,a}^{(0)}],\\ D_3^{(0)} &= 2h(\bar{e}_{31}u_{a,a}^{(0)} - \bar{\varepsilon}_{33}\phi^{(1)}),\end{aligned} \tag{25}$$

$$D_a^{(1)} = -\frac{2}{3}h^3\varepsilon_{11}\phi_{,a}^{(1)}, \tag{26}$$

$$\begin{aligned}J_a^{(0)} &= 2hq(-\bar{n}\mu_{11}\phi_{,a}^{(0)} + n^{(0)}\mu_{11}\overline{E}_a - d_{11}n_{,a}^{(0)}),\\ J_3^{(0)} &= 2hq(-\bar{n}\mu_{33}\phi^{(1)} + n^{(0)}\mu_{33}\overline{E}_3 - d_{33}n^{(1)}),\end{aligned} \tag{27}$$

$$J_a^{(1)} = \frac{2}{3}h^3 q(-\bar{n}\mu_{11}\phi_{,a}^{(1)} + n^{(1)}\mu_{11}\overline{E}_a - d_{11}n_{,a}^{(1)}), \tag{28}$$

where

$$\begin{aligned} &\bar{c}_{11} = c_{11} - c_{13}^2/c_{33}, \quad \bar{c}_{12} = c_{12} - c_{13}c_{32}/c_{33}, \\ &\bar{e}_{31} = e_{31} - e_{33}c_{31}/c_{33}, \quad \bar{\varepsilon}_{33} = \varepsilon_{33} + e_{33}^2/c_{33}, \end{aligned} \tag{29}$$

and $\kappa_1 = \kappa_2 = \kappa$. Substitution of (23)–(28) into (13) yields the following equations for extension

$$\begin{aligned} &\bar{c}_{11}u_{1,11}^{(0)} + c_{66}u_{1,22}^{(0)} + (\bar{c}_{12} + c_{66})u_{2,21}^{(0)} + \bar{e}_{31}\phi_{,1}^{(1)} + \frac{1}{2h}t_1^{(0)} = \rho\ddot{u}_1^{(0)}, \\ &c_{66}u_{2,11}^{(0)} + \bar{c}_{11}u_{2,22}^{(0)} + (\bar{c}_{12} + c_{66})u_{1,12}^{(0)} + e_{15}\phi_{,2}^{(1)} + \frac{1}{2h}t_2^{(0)} = \rho\ddot{u}_2^{(1)}, \end{aligned} \tag{30}$$

flexure

$$\kappa^2 c_{44}(u_{3,aa}^{(0)} + u_{a,a}^{(1)}) + \kappa e_{15}\phi_{,aa}^{(0)} + \frac{1}{2h}t_3^{(0)} = \rho\ddot{u}_3^{(0)} \tag{31}$$

thickness-shear

$$\begin{aligned} &\bar{c}_{11}u_{1,11}^{(1)} + c_{66}u_{1,22}^{(1)} + (\bar{c}_{12} + c_{66})u_{2,21}^{(1)} - 3h^{-2}\kappa^2 c_{44}(u_1^{(1)} + u_{3,1}^{(0)}) - 3h^{-2}\kappa e_{15}\phi_{,1}^{(0)} + \frac{3}{2h^3}t_1^{(1)} = \rho\ddot{u}_1^{(1)}, \\ &c_{66}u_{2,11}^{(1)} + \bar{c}_{11}u_{2,22}^{(1)} + (\bar{c}_{12} + c_{66})u_{1,12}^{(1)} - 3h^{-2}\kappa^2 c_{44}(u_2^{(1)} + u_{3,2}^{(0)}) - 3h^{-2}\kappa e_{15}\phi_{,2}^{(0)} + \frac{3}{2h^3}t_2^{(1)} = \rho\ddot{u}_2^{(1)}, \end{aligned} \tag{32}$$

electrostatics

$$\begin{aligned} &-\varepsilon_{11}\phi_{,aa}^{(0)} + \kappa e_{15}(u_{3,aa}^{(0)} + u_{a,a}^{(1)}) + \frac{1}{2h}d^{(0)} = qn^{(0)}, \\ &-\varepsilon_{11}\phi_{,aa}^{(1)} + 3h^{-2}\bar{\varepsilon}_{33}\phi^{(1)} - 3h^{-2}\bar{e}_{31}u_{a,a}^{(0)} + \frac{3}{2h^3}d^{(1)} = qn^{(1)}, \end{aligned} \tag{33}$$

and conservation of charge

$$\begin{aligned} &\dot{n}^{(0)} - \bar{n}\mu_{11}\phi_{,aa}^{(0)} + n_{,a}^{(0)}\mu_{11}\overline{E}_a - d_{11}n_{,aa}^{(0)} + \frac{1}{2q}j^{(0)} = 0, \\ &\dot{n}^{(1)} - \bar{n}\mu_{11}\phi_{,aa}^{(1)} + n_{,a}^{(1)}\mu_{11}\overline{E}_a - d_{11}n_{,aa}^{(1)} - 3h^{-2}(-\bar{n}\mu_{33}\phi^{(1)} + n^{(0)}\mu_{33}\overline{E}_3 - d_{33}n^{(1)}) + \frac{2}{2h^3 q}j^{(1)} = 0. \end{aligned} \tag{34}$$

5. Propagation of thickness-shear waves

Thickness-shear waves are widely used in plate piezoelectric devices (Wang and Yang, 2000). As an example for the application of the above equations we study the propagation of thickness-shear waves in a plate of 6 mm crystals. These waves are usually accompanied by weak flexural deformations. They are described by $u_a^{(1)}$, $u_3^{(0)}$, $\phi^{(0)}$, and $n^{(0)}$, and are governed by (31)–$(34)_1$ which are not coupled to the other equations.

5.1. Thickness-shear approximation

The weak flexural deformation accompanying thickness-shear can be eliminated by the so called thickness-shear approximation (Tiersten, 1969a) which further simplifies the problem. We consider the case when there are no surface loads. This means that the major surfaces of the plate are traction free, unelectroded without surface free charge, and that the surfaces are perfect without broken surface bonds so that there is no surface charge and current due to semiconduction (Navon, 1986). Enlightened by the thickness-shear approximation for a system of one-dimensional equations for quartz (Tiersten, 1969a), we proceed as follows. From (31) and $(33)_1$, we obtain, by eliminating $\phi^{(0)}$,

$$\kappa^2\left(c_{44}+\frac{e_{15}^2}{\varepsilon_{11}}\right)(u_{3,aa}^{(0)}+u_{a,a}^{(1)})-\frac{\kappa e_{15}q}{\varepsilon_{11}}n^{(0)}=\rho\ddot{u}_3^{(0)}. \tag{35}$$

Substitution of the following wave solution into (35),

$$\begin{aligned} u_3^{(0)} &= A_3\exp[\mathrm{i}(\omega t+\xi_a x_a)],\\ u_b^{(1)} &= A_b\exp[\mathrm{i}(\omega t+\xi_a x_a)],\\ n^{(0)} &= B\exp[\mathrm{i}(\omega t+\xi_a x_a)], \end{aligned} \tag{36}$$

where A_i and B are constants, results in the following relation:

$$\kappa^2\left(c_{44}+\frac{e_{15}^2}{\varepsilon_{11}}\right)(-A_3\xi_a\xi_a+A_a\mathrm{i}\xi_a)-\frac{\kappa e_{15}q}{\varepsilon_{11}}B=-\rho\omega^2A_3. \tag{37}$$

We are interested in long waves with small wave numbers ξ_a. The term quadratic in ξ_a in the above equation can be dropped. Also, since for long thickness-shear waves the frequency ω is very close to the exact thickness-shear frequency of an infinite plate, we make the following substitution in (37):

$$\omega^2\approx\omega_\infty^2=\frac{\pi^2c_{44}}{4\rho h^2}=\frac{3\kappa^2c_{44}}{\rho h^2}, \tag{38}$$

where ω_∞ is the exact infinite plate thickness-shear frequency. Note that (38) does not have the stiffening effect due to piezoelectric coupling. This is because for a plate of 6 mm crystals with the 6-fold axis along the plate normal, exact thickness-shear modes from the three-dimensional equations are elastic only without piezoelectric coupling. In this case the shear correction factor $\kappa^2=\pi^2/12$ (Yang and Zhang, 1999). These lead to the following approximate version of (37):

$$A_3=-\frac{h^2}{3}\left(1+\frac{e_{15}^2}{c_{44}\varepsilon_{11}}\right)\mathrm{i}\xi_aA_a+\frac{4h^2\kappa e_{15}q}{\pi^2c_{44}\varepsilon_{11}}B, \tag{39}$$

which is equivalent to the differential relation

$$u_3^{(0)}=-\frac{h^2}{3}(1+k_{15}^2)u_{a,a}^{(1)}+\frac{4h^2\kappa e_{15}q}{\pi^2c_{44}\varepsilon_{11}}n^{(0)}, \tag{40}$$

where we have denoted

$$k_{15}^2=\frac{e_{15}^2}{c_{44}\varepsilon_{11}}. \tag{41}$$

Substituting (40) into (32) and $(33)_1$, neglecting the third and higher order derivatives of $u_3^{(0)}$ under the long wave approximation, we obtain the following equations under the thickness-shear approximation:

$$\begin{aligned} &c_{11}^*u_{1,11}^{(1)}+c_{66}u_{1,22}^{(1)}+c_{12}^*u_{2,12}^{(1)}-\rho\omega_\infty^2u_1^{(1)}-3h^{-2}\kappa e_{15}\phi_{,1}^{(0)}-\frac{\kappa e_{15}q}{\varepsilon_{11}}n_{,1}^{(0)}=\rho\ddot{u}_1^{(1)},\\ &c_{66}u_{2,11}^{(1)}+c_{11}^*u_{2,22}^{(1)}+c_{12}^*u_{1,12}^{(1)}-\rho\omega_\infty^2u_2^{(1)}-3h^{-2}\kappa e_{15}\phi_{,2}^{(0)}-\frac{\kappa e_{15}q}{\varepsilon_{11}}n_{,2}^{(0)}=\rho\ddot{u}_2^{(1)},\\ &-\varepsilon_{11}\phi_{,aa}^{(0)}+\kappa e_{15}u_{a,a}^{(1)}+k_{15}^2\frac{h^2}{3}qn_{,aa}^{(0)}=qn^{(0)}, \end{aligned} \tag{42}$$

where

$$c_{11}^* = \bar{c}_{11} + \kappa^2 c_{44}(1 + k_{15}^2),$$
$$c_{12}^* = \bar{c}_{12} + c_{66} + \kappa^2 c_{44}(1 + k_{15}^2), \tag{43}$$

and $(34)_1$ remains the same.

5.2. Propagation of thickness-shear waves

We consider waves propagating in the minus x_1 direction with $u_2 = 0$ and $\partial/\partial x_2 = 0$. The biasing electric field is applied in the x_1 direction. Real devices are finite plates. A biasing electric field in the x_1 direction can be produced by side electrodes on the lateral surfaces where $x_1 =$ constant, which is usually called a lateral electric field (Meitzler et al., 1988) in comparison with the thickness field in the x_3 direction. Then (42) and $(34)_1$ reduce to

$$\begin{aligned} &c_{11}^* u_{1,11}^{(1)} - \rho\omega_\infty^2 u_1^{(1)} - 3h^{-2}\kappa e_{15}\phi_{,1}^{(0)} - \frac{\kappa e_{15} q}{\varepsilon_{11}} n_{,1}^{(0)} = \rho\ddot{u}_1^{(1)}, \\ &-\varepsilon_{11}\phi_{,11}^{(0)} + \kappa e_{15} u_{1,1}^{(1)} + k_{15}^2 \frac{h^2}{3} q n_{,11}^{(0)} = q n^{(0)}, \\ &\dot{n}^{(0)} - \bar{n}\mu_{11}\phi_{,11}^{(0)} + n_{,1}^{(0)}\mu_{11}\overline{E}_1 - d_{11} n_{,11}^{(0)} = 0. \end{aligned} \tag{44}$$

Let

$$\begin{aligned} u_1^{(1)} &= A\exp[\mathrm{i}(\xi_1 x_1 - \omega t)], \\ \phi^{(0)} &= B\exp[\mathrm{i}(\xi_1 x_1 - \omega t)], \\ n^{(0)} &= C\exp[\mathrm{i}(\xi_1 x_1 - \omega t)], \end{aligned} \tag{45}$$

where A, B and C are undetermined constants. Substitution of (45) into (44) yields the following linear, homogeneous equation for A, B and C:

$$\begin{bmatrix} \rho(\omega^2 - \omega_\infty^2) - c_{11}^*\xi_1^2 & -3h^{-2}\kappa e_{15}\mathrm{i}\xi_1 & -\dfrac{\kappa e_{15} q}{\varepsilon_{11}}\mathrm{i}\xi_1 \\ \kappa e_{15}\mathrm{i}\xi_1 & \varepsilon_{11}\xi_1^2 & -k_{15}^2\dfrac{h^2}{3}q\xi_1^2 - q \\ 0 & \bar{n}\mu_{11}\xi_1^2 & \mathrm{i}(\xi_1\mu_{11}\overline{E}_1 - \omega) + d_{11}\xi_1^2 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = 0. \tag{46}$$

For nontrivial solutions the determinant of the coefficient matrix has to vanish, which gives the following dispersion relation:

$$\begin{aligned} \rho\omega^2 = \rho\bar{\omega}_\infty^2 + c_{11}^*\xi_1^2 &- \frac{\bar{n} q \mu_{11}}{\varepsilon_{11}[d_{11}\xi_1^2 + \mathrm{i}(\mu_{11}\overline{E}_1\xi_1 - \omega)]} \\ &\times \left\{\left(1 + \frac{1}{3}h^2 k_{15}^2\xi_1^2\right)[\rho(\omega^2 - \omega_\infty^2) - \xi_1^2 c_{11}^*] + \kappa^2 k_{15}^2 c_{44}\xi_1^2\right\}, \end{aligned} \tag{47}$$

where

$$\rho\bar{\omega}_\infty^2 = \rho\omega_\infty^2 + \rho\frac{3\kappa^2 c_{44}}{\rho h^2}\frac{e_{15}^2}{\varepsilon_{11} c_{44}} = \rho\omega_\infty^2(1 + k_{15}^2), \tag{48}$$

is the piezoelectrically stiffened infinite plate thickness-shear frequency when there exists a coupling to an electric filed in the x_1 direction. The denominator of the right-hand side of (47) indicates that wave amplification may occur when $\mu_{11}\overline{E}_1\xi_1 - \omega$ changes it sign or

$$\frac{\omega}{\xi_1} = \mu_{11}\overline{E}_1, \tag{49}$$

i.e., the acoustic wave speed is equal to the carrier drift speed (White, 1962). When the semiconduction is small, (47) can be solved by an iteration or perturbation procedure. As the lowest (zero) order of approximation, we neglect the semiconduction and (47) reduces to the dispersion relation of piezoelectric thickness-shear waves

$$\rho\omega_{(0)}^2 = \rho\bar{\omega}_\infty^2 + c_{11}^*\xi_1^2, \tag{50}$$

which is dispersive but not dissipative. For the next order we substitute (50) into the right-hand side of (47) and obtain

$$\begin{aligned}\rho\omega_{(1)}^2 = \rho\bar{\omega}_\infty^2 + c_{11}^*\xi_1^2 - &\frac{\bar{n}q\mu_{11}}{\varepsilon_{11}[d_{11}\xi_1^2 + i(\mu_{11}\overline{E}_1\xi_1 - \omega_{(0)})]} \\ &\times\left\{\left(1+\frac{1}{3}h^2k_{15}^2\xi_1^2\right)[\rho(\omega_{(0)}^2 - \omega_\infty^2) - \xi_1^2c_{11}^*] + \kappa^2k_{15}^2c_{44}\xi_1^2\right\},\end{aligned} \tag{51}$$

which is dispersive and dissipative.

5.3. Numerical results

For numerical results we consider CdS with (Auld, 1973; Gualtieri et al., 1994)

$$\begin{aligned}&\rho = 4820\ \text{kg/m}^3,\\ &c_{11} = 9.07,\quad c_{33} = 9.38,\quad c_{44} = 1.504,\quad c_{12} = 5.81,\quad c_{13} = 5.10\times10^{10}\ \text{N/m}^2,\\ &e_{15} = -0.21,\quad e_{31} = -0.24,\quad e_{33} = -0.44\ \text{C/m}^2,\\ &\varepsilon_{11} = 9.02\varepsilon_0,\quad \varepsilon_{33} = 9.53\varepsilon_0,\quad \varepsilon_0 = 8.854\times10^{-12}\ \text{F/m}.\end{aligned} \tag{52}$$

The mobility of electrons and holes of CdS at 300 K are (Navon, 1986)

$$\mu_n = 340,\quad \mu_p = 50\ \text{cm}^2/\text{V s}. \tag{53}$$

We consider electrons with μ_n. The diffusion constants can be determined from the Einstein relation (Navon, 1986)

$$D = \frac{kT}{q_e}\mu, \tag{54}$$

where T is the absolute temperature, and k the Boltzmann constant. At room temperature $kT/q_e = 0.026$ V (Navon, 1986) where $q_e = 1.602\times10^{-19}$ C is the electronic charge. Present technology can make a material with $\bar{n}$ of any value between an insulator and a conductor. We use $\bar{n} = 10^{13}/\text{m}^3$ which is considered small in the sense that the conduction term in (51) is much smaller than other terms so that the perturbation solution is valid.

We plot the real parts of $\omega_{(0)}$ and $\omega_{(1)}$ versus ξ_1 in Fig. 2. The dimensionless wave number X and the dimensionless frequency Y of different orders are defined by

$$X = \xi_1\Big/\frac{\pi}{2h},\quad Y_{(0)} = \omega_{(0)}/\bar{\omega}_\infty,\quad Y_{(1)} = \text{Re}\{\omega_{(1)}\}/\bar{\omega}_\infty. \tag{55}$$

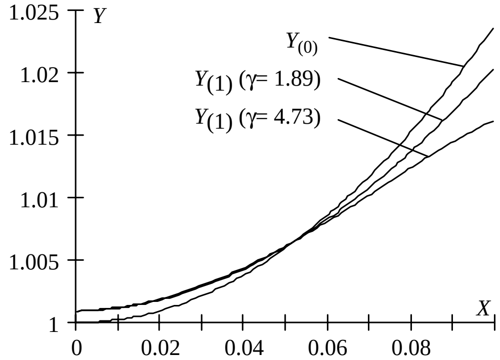

Fig. 2. Dispersion relations of thickness-shear waves; $X=$ dimensionless wave number; $Y=$ real part of the dimensionless frequency; $\gamma=$ normalized biasing electric field.

γ is a dimensionless number given by

$$\gamma = \mu_{11}\overline{E}_1 \bigg/ \frac{\bar{\omega}_\infty 2h}{\pi}, \tag{56}$$

which may be considered as a normalized electric field. It represents the ratio of the electron drift velocity and a quantity related to the acoustic wave speed. In applications long thickness-shear waves with a small X are useful. When $X=0$ thickness-shear waves have a nonzero cutoff frequency which is normalized to one in the figure. The behavior of thickness-shear waves in a piezoelectric insulator is described by $Y_{(0)}$ which is dispersive. It can be seen that semiconduction causes additional dispersion and a shift of the cutoff frequency. This conduction induced dispersion and frequency shift vary according to the dc electric biasing field.

Fig. 3 shows the imaginary part of $\omega_{(1)}$ versus γ which represents $\overline{E}_1$. $\lambda = 2\pi/\xi_1$ is the wavelength. The dimensionless number describing the decaying behavior of the waves is defined by

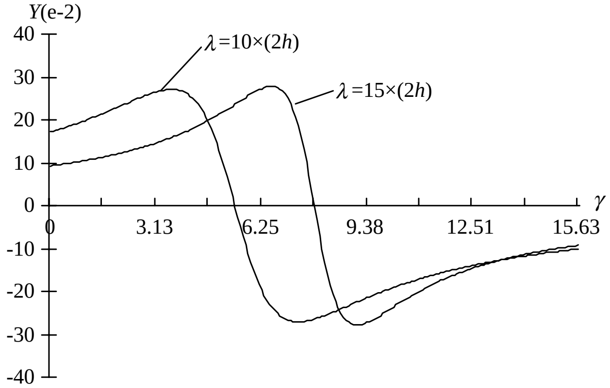

Fig. 3. Dissipation as a function of the dc bias; $Y=$ imaginary part of the dimensionless frequency; $\gamma=$ normalized biasing electric field; $\lambda = 2\pi/\xi_1$.

$$Y = \mathrm{Im}\{\omega_{(1)}\}/\bar{\omega}_{\infty}. \tag{57}$$

When the dc bias is large enough the decay constant becomes negative indicating wave amplification. The transition from damped waves to growing waves in (51) indeed occurs when (49) is true. These agree qualitatively with the behavior of the plane waves studied by White (1962).

Experimental results for acoustoelectric amplification of acoustic waves are available, e.g., surface waves by Collins et al. (1968) and extensional waves in a plate by Dietz et al. (1988). Experimental study for the amplification of the thickness-shear waves in a plate discussed in this paper can be performed in a way similar to Dietz et al. (1988) where a lateral biasing electric field (Meitzler et al., 1988) is used. This is out of the scope of the present paper.

6. Conclusion

Two-dimensional equations for coupled extensional, flexural and thickness-shear motions of thin plates of piezoelectric semiconductors under a dc field are obtained. The equations are specialized to crystals of 6 mm symmetry. It is shown that semiconduction causes acoustic dispersion and loss in the propagation of thickness-shear waves. The equations are useful in analyzing plate structures for devices, and surface waves guided by thin films in the manner of Tiersten (1969b).

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